Chapter 1

MULTIGRAPH DEPENDENCY MODELS FOR HETEROGENEOUS CRITICAL INFRASTRUCTURES

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Abstract The identification and mitigation of interdependencies among critical infrastructure elements such as telecommunication, energy, and transportation are important steps of any critical infrastructure protection strategy and are applicable both in preventive and operative settings. In this paper we present a graph-theoretical model and framework for the analysis of such dependencies based on a multigraph approach and present selected algorithms for the automatic identification of critical dependencies. These algorithms are then applied to dependency structures which simulate the scale-free structures commonly found in many infrastructure networks as well as to networks augmented by random graphs.

Keywords: Interdependency modeling of critical infrastructures, multigraph model, interdependency simulation, infrastructure topology

1. Introduction

One of the defining characteristics of critical infrastructures is the level of interdependence among individual infrastructure components such as energy, telecommunications, and financial services. While the interdependencies act on different timescales and may exhibit buffering characteristics (e.g. in case of emergency power supplies) or delays in the effects (e.g. an inability to schedule transportation services in case of communication system failure) in some instances, direct and transitive (often also circular interdependencies) can be identified in a large number of cases.

An area of particular interest in critical infrastructure protection research is the avoidance and analysis of widespread effects on large parts of the population and economies, which may e.g. result from cascading and circular effects among infrastructure components – as exemplified by the extensive August 2003 power outages in the northeastern U.S. and Canada and the November 2006 power outages throughout much of continental Europe.

While elaborate models, also incorporating physical characteristics and effects and with predictive capabilities exist for many of the individual critical infrastructure services such as for electrical power grids at national and transnational levels, it is clearly desirable to also investigate larger-scale interactions among multiple infrastructure sectors. Specific questions might for example include cascading effects that would occur if one infrastructure component becomes unavailable for extended periods, along with possible circular effects that might inhibit or at least severely impede the resumption of regular infrastructure services. This, however, requires the development of models that exhibit acceptable computational complexity and at the same time provide adequate modeling capabilities. The level of detail which can be incorporated in such models is of necessity a limited one compared to sector-specific models; however, in many cases the basic identification of the existence of interdependencies and critical dependency paths among infrastructure components already provides valuable information which may subsequently be investigated further through more refined modeling processes.

In this paper, we present a model framework based on a simple graph-theoretic model that forms the basis of several models of increasing capabilities (and hence computational complexity) in which additional constraints on the graph model are introduced and infrastructure characteristics such as the ability to buffer resources on which the provision of infrastructure services depend are added (see section 1.6 for a description of ongoing work on models incorporating these additional parameters). Connectivity-based interdependency models, however, can provide important insights into the vulnerabilities introduced by interlinking infrastructure components, particularly if the interdependency characteristics differ significantly as in the case of power and telecommunication networks discussed in the abstract scenario as described in section 1.3.

The remainder of this paper is structured as follows: Section 1.2 provides a concise summary of the basic multigraph model which forms the foundation for a family of models with increasing expressiveness and computational complexity. Section 1.3 then provides several simplified case studies. However, it should be noted that the model instances and scenarios in section 1.3 are primarily intended to be illustrative and hence represent simplified abstractions, not actual network structures. These model instances are subsequently further illustrated through simulation results described in section 1.4. Section 1.5 then briefly reviews related research activities, and section 1.6 provides conclusions and an outlook on current and ongoing research.

2. Multigraph Model

Interactions among infrastructure components and infrastructure users are modeled in the form of directed multigraphs, which can be further augmented by response functions defining interactions between components. In the model, the vertices $\mathcal{V} = \{v_1, \ldots, v_k\}$ are interpreted as producers and consumers of m different types of services. A single vertex can act both as a producer and a consumer at the same time. If a node is not able to generate a needed type, the node is dependent on some other node delivering this service. Such a dependability is named dependability type d_j, and is chosen from the set $\mathcal{D} = \{d_1, \ldots, d_m\}$.

Pairwise dependencies between nodes are represented with directed edges, where the head node is dependent on the tail node. The edges of a given infrastructure are defined by a subset $\mathcal{E}$ of $\boldsymbol{\mathcal{E}} = \{e_1^1, e_2^1, \ldots, e_{n_1}^1, e_1^2, , \ldots, e_{n_m}^m\}$, where $n_1, \ldots, n_m$ respectively are the numbers of dependencies of type $d_1, \ldots, d_m$, and e_i^j is the edge number i of dependency

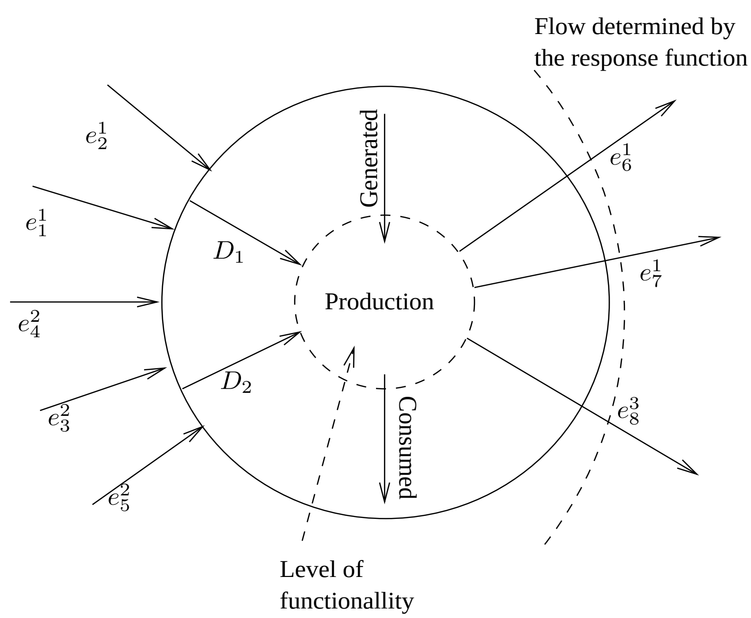


Figure 1. The parameters that define the functionality of a node, and its outputs

type j in the network. A given dependency between two nodes v_a and v_b is uniquely determined by $e_i^j(v_a, v_b)$.

In addition to the type, two predicates $C_{\text{Max}}(e_i^j(v_a, v_b)) \in \mathbb{N}_0$ and $C_{\text{Min}}(e_i^j(v_a, v_b)) \in \mathbb{N}_0$ are defined for each edge. These values respectively represented the maximum capacity of the edge $e_i^j(v_a, v_b)$ and the lower threshold for flow through the edge.

The first studies of large complex networks evaluated the robustness of attacks towards the infrastructure based on static failures [9, 6]. That is removing a certain percentage of the nodes in the network and estimating how the performance or connectivity of network is affected by the induced failure. In dependency networks, as the power distribution network and the telephony transport network, the breakdown of a node may cause cascading failures and have other time-dependent dynamic effects through the network detectable only through a dynamic approach to the networks. We assume the time to be discrete in this model instance and that the system is initiated to an initial state at time $t = 0$.

Let $r_a^j(t) \in \mathbb{Z}$ be the amount of resource j produced in node v_a at time t. We define $D(t)$ to be a $k \times m$ matrix over $\mathbb{Z}$ describing the amount of resources of dependency type j available at the node v_a at time t. It follows that the initial state of D is given by

$$D_{aj}(0) = r_a^j(0). \tag{1}$$

For every edge in $\mathcal{E}$ a response function

$$\begin{aligned} R_i^j(v_a, v_b, t) \;\; = \;\; & f(D_{a1}(t-1), \ldots, D_{am}(t-1), \\ & C_{\text{Max}}(e_i^j(v_a, v_b)), C_{\text{Min}}(e_i^j)(v_a, v_b)) \end{aligned} \tag{2}$$

that determines the i-th flow of type j between the nodes v_a and v_b is defined. The function f w.l.o.g. is defined as a linear function mapping $\mathbb{Z} \times \cdots \times \mathbb{Z} \times \mathbb{N}_0 \times \mathbb{N}_0$ to $\mathbb{N}_0$ (see below for a rationale for limiting f to linear functions), and may contain some prioritizing scheme over i and v_b. As seen from Equation 2 a single step model with one state memory has been chosen, as we are currently not concerned with long term feedback, although the model naturally extends to longer-term state retention.

Given the responses at time t, the available resources in a node v_a at time t in any node are given by

$$D_{aj}(t) = \sum_{i,s \mid e_i^j(v_s, v_a) \in \mathcal{E}} R_i^j(v_s, v_a, t). \tag{3}$$

A node v_a is said to be functional at time t if it receives or generates the resources needed to satisfy its internal needs, that is $D_{aj}(t) > 0$

for all dependency types j which are such that $e_i^j(v_b, v_a) \in \mathcal{E}$, where $b \in \{1, \ldots, a-1, a+1, \ldots k\}$. If this is the case for only some of the dependency types the node is said to be partially functional, and finally of no requirement are satisfied the node is said to be dysfunctional.

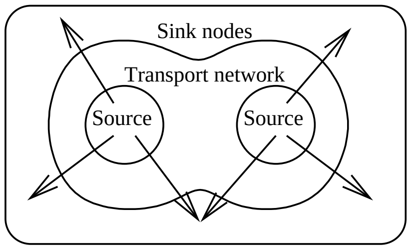
The implemented model investigates how high-level network effects (functionality of nodes) and interrelations (connectivity of nodes) in interconnected infrastructures reacts to different attack scenarios. The presented model can be used to represent any topology given a set of infrastructures and their interconnections. The model cannot achieve the level of accuracy found e.g. in devoted network simulators as described in section 1.5, but has the advantage of being able to estimate the consequences of cascading failures through interconnected infrastructures.

By constraining the response function to a linear function and discrete values for both time steps and resources, linear programming approaches can be employed for optimization of the relevant parameters; interior point methods for this type of problem such as [18, 24] can achieve computational complexity on the order of $O(n^{3.5})$, making the analysis of large graphs feasible.

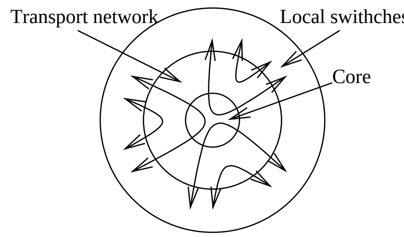
3. Dependency Analysis

This section explores how two interconnected networks influence each other. Two clearly interdependent networks are the power distribution network and the telephony transport layer. The analysis is based on several abstractions and represents an approximation to actual network topologies. The motivation for choosing these two classes of infrastructure elements as the first subject of investigation is due primarily to their key enabling role in industrial societies. The BAS study [16], carried out by the Norwegian Defense Research Establishment in 1997, established the criticality of power-supply and telecommunication in the Norwegian society. These networks are therefore natural candidates for such an analysis and have been subject of separate investigations (see section 1.5).

In addition the networks are interesting candidates for model verification, as there is a fundamental difference in how service deliveries flow through the networks. In the power distribution network all the generated power originates from a small number of power plants or generators. A transportation network, which may well interconnect several power plants, delivers the power to a large number of transformers, which server the low voltage distribution network. This is illustrated in figure 2(a). As a consequence the resulting graph is a directed network



(a) Flow in the power distribution network.



(b) Flow in the telephony transport layer.

Figure 2. Illustration of the difference of flow though the two considered networks

where multiple edges of different orientation between two nodes rarely occurs.

Traditionally the telecommunication layer has been a hierarchical network, as described e.g. in [14]. Although there has been a decided trend away from this due to progress in transmission and switching technology since the early 1990s, we chose to use this model since it is representative of much of the currently deployed telecommunication infrastructure. The telecommunication transport layer could be described as an onion structure with a very low diameter. The signal always starts from the outer layer, then depending on the range of the connection, it goes through the core of the network before retuning to a local switch in the outer layer of the network. All edges are bidirectional as illustrated in figure 2(b). As the flow through these edges is bidirectional all connected nodes will be connected by an edge in each direction.

3.1 Electric Grid Network

Recently extensive studies of the power distribution has been carried out in the open literature. One of the early analysis was of the Western States Power Grid (US) carried out by Watts and Strogatz [27] in 1998. The degree distribution of the network was found to be exponential-like, but the clustering coefficients are too large for the network to be a classical random graph. The observed network consisted of approximately 3500 nodes, a number which might be too small for being conclusive regarding the categorization of the network [11]. For the purposes of the present study, however, an exact representation of the power distribution is not necessary as we are primarily interested in topological characteristics. To this end, a network topology generator was implemented based on the following assumptions:

1 The number of source nodes is small compared to the number of transport and sink nodes in the network.

2 Power generating nodes are not directly interconnected.

3 The network is constructed in order to cover a topological area as efficiently as possible.

4 Some redundant links are forced on the network in order to interconnect distribution networks and create redundancy.

Based on this, a tree-like model for the power distribution network seems reasonable as an approximation, although we find binary or k-trees too regular to represent the topology. The basic Barabási-Albert (BA) model [1] with some modifications provides a tree-like structure together with the level of irregularity found in real networks. The original BA model is initiated with a connected graph. In the power distribution network case, the source nodes are not interconnected. This is solved by simply providing the originating nodes with an initial degree $k_{\mathrm{Init}} \geq 1$ which does not represent any real edges, just the centrality of the node in the network.

Given that one node is added at each time step in the BA-model, as many disconnected trees as there are initial nodes in the network will be generated. In order to connect lover level nodes with each other a sparse random graph is placed on the top of the scale free networks. Given that this is a very sparse network it will not affect the statistical properties of the network, but is has a major influence on the connectivity of the network and may also generate feedback loops.

The following procedure is used to generate the power distribution network topology.

1 Growth: At every time step a new node is added to the network. This node defines the head of an edge connecting it to an already existing node.

2 Preferential attachment: The tail of the edge is selected among the existing nodes with probability proportional to the degree of the node.

3 Redundant connection: After the final time step a sparse random graph is placed on the top of the network.

As the network grows large, the influence of the sparse random graph will be small, and the probability of a node having k edges will follow a power law with exponent $\gamma = 3$.

Finally the response function for each edge is defined. In the case of quantitative analysis of service delivery this function should be an implementation of Kirchhoff's first rule, ensuring that all the flow into a node together with the flow generated by a node equals the output and the consumption of the node. In order to explore the presented model such an detailed approach is not necessary, as the model focuses on the functionality of the node. The principal issue is that the electricity is consumed as it propagates through the networks and cannot e.g. be stored in subgraph cycles. Thus the implemented response function only illustrates a resource which is being consumed as it flows through the network. Introducing a threshold function

$$T(x,c) = \delta(x-c)x, \tag{4}$$

where

$$\delta(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0. \end{cases} \tag{5}$$

the implemented response function is on the form

$$R_i(v_a, v_b, t)) = T(\frac{1}{2}D_a(t), C_{\mathrm{Min}}(e_i(v_a, v_b))), \tag{6}$$

where D_a is the current available in the node a at time t. Equation 6 indicates that two units of input current to the node is required to produce one unit of output current along an outgoing edge. As there is only one dependency in the network the dependency type is not specified. We assume that there exists only one power dependency between two nodes and no prioritization scheme is defined over the outgoing edges.

A node in the power distribution network is defined to be functional if it is has incoming current or generates current internally. The given response function can provide information on whether a node is functional or not, but does not provide any physical representation of the level of functionality of a given node in the network, which provides a sufficient level of details for the purposes of the present study.

3.2 Telephony Transport Layer

Compared to the electric grid, telephony transport layer has received much less attention in the scientific community modeling critical infrastructures. Whereas the Internet and autonomous system networks have been modeled extensively [21], modeling the telephony transport layers functionality and design is left to operators. As mentioned in the introduction to this section, we assume a telephony transport layer having a traditional hierarchical network structure. This is a network which is optimized locally for full connectivity, and globally to reduce the number of switched included in an average connection circuit. In order to be functional a switch needs to be connected to other switches and to power supply, which is the focus of the following analysis.

The network is modeled as a number of disconnected trees which is connected to a fully connected transportation network through their root nodes. The response function of the telephony network depends on whether the node has power as input or not. If there is no power available circuit switching cannot take place and no communication is possible. In the implemented model the response function for edges in the telephony transport layer is thus a threshold function given by

$$R_i(v_a, v_b, t) = \delta(D_a(t) - C_{\text{Min}}(e_i(v_a, v_b))), \tag{7}$$

where D_a is the current available in the node a at time t and δ is as defined in Equation 5. It follows from the definition of equation 2 that a directed edge between the nodes v_a and v_b is defined if there is power available in node v_a. Again, no redundant links are defined between two nodes and no prioritization scheme is defined over the edges.

As mentioned earlier, each connection in the telephony transport layer is bidirectional, as one way communications are of no interest. The functionality of a node thus depends on if the node itself and the node it is connected has power supply. Only in this case the switch can deliver the two way service it is meant to deliver.

3.3 Interconnection of Power Distribution and Telephony Networks

The dependency between the power distribution network and the telephony transport layer is assumed to be one way. This implies that the modeled power distribution network can be fully functional even if no switches in the telephony transport layer is functional. Conversely the flow along an edge in the telecommunication network will halt if either the head node or tail node looses its power supply. The connection of the telephony transport layer to the power grid is randomized in the present model (i.e. does not take into account geospatial proximity or other factors that would otherwise result in functional clustering). However, for the purposes of the present analysis, this is considered adequate.

The nodes of the telephony transportation layer now have two inputs, current and information, and give output in form of information. At every time step the response functions for power distribution and telephony transportation edges can be computed over their respective domains given the network state in the previous time step. From this the functionality of the telephony transport layer follows directly.

Since only a one-way dependency has been defined, failure can only propagate from the power distribution network into the telephony transportation layer, although the present paper only investigates a constrained class of dependencies.

3.4 Attack Scenarios

The studies of complex networks frequently conclude that many man made and natural networks are scale free of nature, and thus possess the well known Achilles heel of robustness against random breakdown and vulnerability towards targeted attacks [2]. The first item investigated in section 1.4 is that the introduction of a very sparse random graph on the top of a scale free infrastructure will reduce some of the networks vulnerability for targeted attacks.

There are many possible scenarios that may cause the failure of a node in some infrastructure. The cause may be intentionally or unintentionally created by humans or they may be the consequence of some changes in the environment of the network, e.g. flooding and temperature changes, or by technical errors leading to an failure. Analyzing the presented network models we have the following three attack scenarios in mind:

1 Single node removal: This can be the consequence of a targeted terrorist attack or single technical failure.

2 Removal of small connected component: Representing non-localized failures such as flooding or other natural disasters.

3 Removal of disconnected components: This could e.g. be the result of a coordinated terrorist attack.

These attack scenarios are further analyzed in section 1.4.

4. Simulations

To illustrate the properties of the presented model, artificially small topologies were generated. A power distribution topology based on two power sources and 28 power distribution nodes were connected to a telephony transport network with three core switches and a total of 21 switches. The switches were connected to randomly selected lower level power distribution nodes (meaning that no power generating nodes were connected directly to the telephony transport layer). None of the nodes of the telephony transport layer were assumed to have an independent power supply.

For all the presented scenarios two attacks are considered. Random node removal follows the procedure

1 Remove a node from the network

2 Run the response function until the number of functional nodes in the network stabilizes.

3 Count the number of functional nodes in the network

4 Reinsert the node

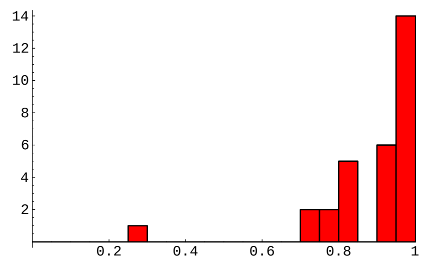
The procedure is repeated for all nodes in the network. Pairwise removal of nodes, follows the same procedures, but in this case two nodes are removed from the graph at the time. Results of the attacks are presented as fraction of functional remaining nodes after the removal of one or two nodes from the network. the results are presented as histograms in figure 3, 4, and 5. The x-axis represents the fraction of functional nodes in a run, and the y-axis represents number of runs.

The presented results are deduced from one topology generated as described in section 1.3. A single topology is not sufficient to draw any general conclusions on the properties of the proposed topologies, but illustrates the ability and flexibility of the presented model.

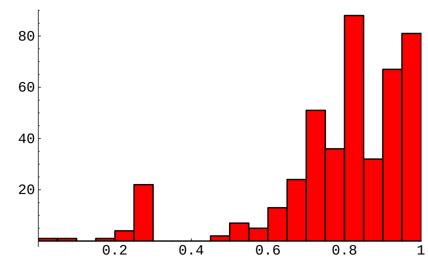
4.1 Coordinated Failures within a Single Domain

In this scenario, the single, non-buffered power distribution network is considered. While atypical of the interdependencies between infrastructure types encountered in the field of critical infrastructure, this permits the exposition of core elements of the model and simulation environment.

4.1.1 Scale-Free Power Distribution Network. This scenario illustrates the well known vulnerability of scale free networks towards targeted attacks. The power distribution network is represented as a scale free network. Two scenarios are considered. Single node removal, and removal of two randomly selected power nodes. The results of these attacks are shown in figure 3, and nicely illustrates the properties of scale free network. Figure 3(a) shows that removal of one node in the majority of the cases has limited influence on the network. In almost 50% of the cases the functionality of the nodes in the network will remain above 95% (which is to be considered very high, as the simulated power distribution network has 28 nodes). We also note the high influence of the removal of one node. This is the generator in the largest

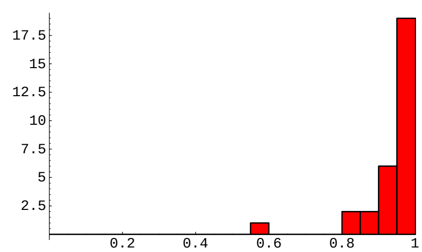


(a) Remaining fraction of functional power nodes after random removal of one power node (28 runs of the algorithm)

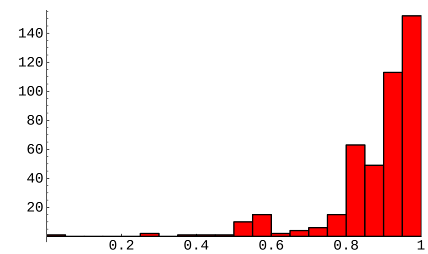


(b) Remaining fraction of functional power nodes after random removal of two power nodes (378 runs of the algorithm)

Figure 3. The consequences of one and two node removals on a scale free topology with 28 nodes.



(a) Remaining fraction of functional power nodes after random removal of one power node (51 runs of the algorithm)



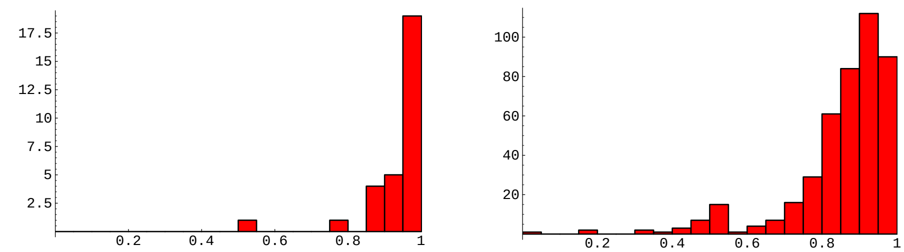
(b) Remaining fraction of functional power nodes after random removal of two power nodes (1275 runs of the algorithm)

Figure 4. The consequences of one and two node removals from a scale free network with added redundancy.

sub-distribution network. As the distribution networks of the two generators are not interconnected due to the BA construction, this takes out the entire subgraph. The gap observed between 0.85% and 0.90% of functional nodes is most likely due to the small size of the network.

Figure 3(b) shows that effect of removal of two nodes from the network also is limited. After all the functionality of the nodes remain above 70% in the majority of the cases. Obviously taking out both generators paralyzes the network. The peak observed around 30% are due to the removal of the largest generator plus a central node in the second power distribution network.

4.1.2 Scale Free Power Distribution Network with Added Redundancy. In this scenario a sparse random graph, as discussed



(a) Remaining fraction of functional telecommunication nodes after random removal of one power node (51 runs of the algorithm)

(b) Remaining fraction of functional telecommunication nodes after random removal of two power nodes (1275 runs of the algorithm)

Figure 5. The consequences of one and two node removals on the two networks.

earlier, is placed on the top of the scale free graph to provide redundancy. The results of the simulations are shown in figure 4.

figure 4(a) shows that the introduced redundancy gives a considerable improvement to the robustness of the network. In the case of one node removal the functionality of the network rarely drops below 90%, and never below 50%. Thus the cost of adding redundant edges, may pay of in terms of robustness.

The same goes for the scenario with two node removal, results shown in figure 4(b). The functionality rarely drops below 80% and the peak earlier located around 30% has now moved to 60%. Of course the removal of both generators still takes out the entire network.

An interesting observation was made related to the second attack scenario described in section 1.3.4. In the 15 most critical removals of two nodes there were no pair of connected nodes. So removing any connected component of size two, still resulting in more than 50% of nodes remaining functional. This shows that well targeted attacks are likely to be more effective towards critical infrastructures than an extensive attack against connected components.

4.2 Multi-Domain Dependencies

The final simulations illustrates how failures in the power distribution network propagates into the telephony transport layer. Each node of the telephony transport layer is connected to a node in the power distribution network, and its functionality is depended on the power supply of itself and of its neighbors. Figure 5 shows the fraction of fully functional telecommunication nodes as one or two nodes are being removed from the power distribution network.

The results clearly illustrates the dependency between the two networks and validates the basics of the model. We refer to future work to explore more exiting features of the model such at circular dependencies, multiple network interdependencies and suitable metrics for identification of critical network components.

4.3 Results of the simulations

The first result lies in the generation of the network topologies. Pure scale free topologies are in our opinion not suitable as representation for real world infrastructure. The pure BA topology contains very few redundant links, while as man made infrastructure in general is supposed to contain at least some dependency. Imposing some random connections on the top of the BA structure brings the model closer to such a realistic constraint. At the same time it can be observed that the vulnerability of the networks are brought down. Redundancy is introduced and the network is less sensitive to the removal of single nodes. This is illustrated in Section 1.4.

Further results of the analysis are to consider removal of random nodes and random pairs of nodes from the network, and observe failure propagation through the network. This can give indications on which nodes that are central for the functionality of the network and where resources should be invested to increase both operational reliability and security at the facilities.

5. Related Work

Research activities regarding the monitoring and simulation of critical infrastructures are being conducted worldwide, although generally at a qualitative level. Among the earliest and most widespread is the application of a control systems approach [25] including hybrid mechanisms [17]. Other approaches that have been investigated for modeling infrastructures include agent-based systems [4, 20, 26]. Such qualitative efforts also include the Critical Infrastructure Modeling and Assessment Program (CIMAP) by Rahman *et al.* and the European Project ACIP [23]. Additional approaches also include [3] and [22] which vary considerably in the level of detail considered, ranging from simple dependency analyses to elaborate models containing continuous physical submodels (e.g. for pipelines and electrical grid systems) as well as behavioral models.

For the more constrained case of individual infrastructures such as electrical and pipeline grid environments, however, rich modeling and simulation environments already exist including the PSIcontrol system and proprietary mechanisms employed by grid operators. Interconnections and interdependencies can only be modeled to a limited extent in such environments. Several properties are immediately derivable from interconnection characteristics alone as shown for power grid and Internet connectivity by Faloutsos *et al.* [13], Broder *et al.* [5], Yook *et al.* [28], and Chen *et al.* [8]. Frequently, the underlying structure of the networks can be identified as being wholly or partially scale-free [12, 15, 7, 19]. This has significant implications for the vulnerability of interconnected and interdependent networks of critical infrastructure components to random failure [6, 9], but also to targeted attacks [10].

6. Conclusions

In this paper we have presented the foundational elements of a family of models for investigating interdependencies among heterogeneous critical infrastructures. To this end, we have provided an extensible graph-theoretical model which incorporates the use of a flexible response function to accommodate the modeling of vertex behavior, including of activities internal to vertices and also of unbuffered and buffered infrastructure service provision.

Based on simplified abstract models we have subsequently illustrated how the addition of random component to an otherwise scale-free network can influence the overall robustness of the network to vertex removal and provide simulation results for scenarios based on a simple interconnection model for two unbuffered networks, i.e. abstract representations of fixed-line telephony network and the electrical grid.

Future and ongoing work will include the validation of the simulations against large-scale power and telephony network topology data, but is particularly focused on providing extensions to the model in which the response function can accommodate multiple resources being provided by each individual vertex in both discrete and continuous variables, resulting in a web of interdependencies, as well as the development of optimization techniques for increasing the robustness of critical infrastructures.

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