Dissertation Title

Dissertation Subtitle

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Dissertation Title

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Sometimes a scream is better than a thesis.

(Ralph Waldo Emerson)

Declaration of Authorship

I, Stephen D. Wolthusen, hereby declare that this thesis and the work presented in it is entirely my own. Where I have consulted the work of others, this is always clearly stated.

Signed:

(Stephen D. Wolthusen)

Date:

Preface

The dissertation preface. It should be kept very short, typically 1–3 paragraphs.

Summary

This file contains the abstract or summary of the dissertation with key contributions described in the space of 1–2 pages.

Acknowledgments

De nihilo nihil.

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Chapter 1

Introduction

> Young men should prove theorems, old men should write books.
>
> G. H. HARDY

The Pregel River flowing through the city of Königsberg contained two islands. Both sides of the mainland were connected one side of the mainland with one island with a third bridge joining the same side of the mainland with the other island and a further bridge between the two islands. Leonhard Euler (1707–1783) formalised the problem whether it was possible to cross each bridge (see figure 1.1 on the following page) exactly once during a single traversal [1].

Definition 1.1 (Graphs)
A graph $G = (V, E)$ *is an ordered pair of finite sets. Elements of* V *are called* vertices *or* nodes, *and elements of* $E \subseteq V^{(2)}$ *are called* edges *or* arcs. V *is referred to as the* vertex set *of* G *while* E *is referred to as the* edge set. *The cardinality of* V *is called the* order *of* G, *and* $|E|$ *is called the* size *of* G.

Unless noted otherwise, directions of edges are not considered with (u, v) *and* (v, u) *regarded as the same edge in* G. *In that case,* G *is referred to as an* undirected graph.

Euler was born in Basel, Switzerland, and had studied mathematics under Johann Bernoulli (1667–1748). He was extremely prolific, contributing to a variety of subjects ranging from number theory and analysis to astronomy, but also to topology and graph theory.

Definition 1.2 (Connected Graphs)
A connected graph *is a graph such that there exists a path between all pairs of vertices. If the graph is a directed graph, and there exists a path from each vertex to every other vertex, then it is a* strongly connected graph.

Theorem 1.1 (Euler Circuit)
A finite graph G *contains an Euler path if and only if* G *is connected and contains no vertices of odd degree.*

Corollary 1.2 (Euler Path)
A finite graph G *contains an Euler path if and only if* G *is connected and contains at most two vertices of odd degree.*

The identification of pertinent literature is crucial [4, 3], but for the purposes of a thesis template, it is more pertinent to study the properties of different types

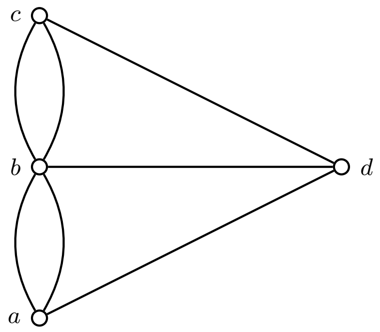

Figure 1.1: The seven bridges of Königsberg

of publications and their citation [2], particularly with regard to the possibility of including DOI. Similar issues also arise when introducing spurious nomenclature such as power sets.

A *bipartite graph* G is a graph with at least two vertices such that $V(G)$ can be split into two disjoint subsets V_1 and V_2, both nonempty. Every edge $uv \in E(G)$ is such that $u \in V_1$ and $v \in V_2$, or $v \in V_1$ and $u \in V_2$.

Theorem 1.3 (Bipartite Graph)
A graph is bipartite if and only if it has no odd cycles.

PROOF ($\Longrightarrow$): Assume G is bipartite. Traversing each edge requires going from one side of the bipartition to the other. For a path to be closed, it must have even length in order to return to the side of the bipartition from which the path started. Thus, any cycle in G must have even length.

($\Longleftarrow$): Assume $G = (V, E)$ has order $n \geq 2$ and no odd cycles. If G is connected, choose any vertex $u \in V$ and define a partition of V thus:

$$X = \{x \in V \mid d(u, x) \text{ is even}\},$$
$$Y = \{y \in V \mid d(u, y) \text{ is odd}\}$$

where $d(u, v)$ denotes the distance (or length of the shortest path) from u to v. If (X, Y) is a bipartition of G, nothing further is needed. Otherwise, (X, Y) is not a bipartition of G. Then one of X and Y has two vertices v, w joined by an edge e. Let P_1 be a shortest u-v path and P_2 be a shortest u-w path. By definition of X and Y, both P_1 and P_2 have even lengths or both have odd lengths.

From u, let x be the last vertex common to both P_1 and P_2. The subpath u-x of P_1 and u-x of P_2 have equal length. That is, the subpath x-v of P_1 and x-w of P_2 both have even or odd lengths. Construct a cycle C from the paths x-v and x-w, and the edge e joining v and w. Since x-v and x-w both have even or odd lengths, the cycle C has odd length, contradicting our hypothesis that G has no odd cycles. Hence, (X, Y) is a bipartition of G.

Finally, if G is disconnected, each of its components has no odd cycles. The above argument is repeated for each component to conclude that G is bipartite. ■

It may also be necessary to include algorithms in the course of the dissertation; in this case pseudo-code is typically to be preferred as can be seen in algorithm 1.1 on the next page.

Graph Name
Bipartite graph
Chvátal graph
Complete graph
Random graph

Table 1.1: Various graph types

Similarly, tables such as the one shown in table the completely gratuitous table shown in table 1.1.

Algorithm 1.1: Generating a random graph in $\mathcal{G}(n, p)$.

Input : Positive integer n and a probability $0 < p < 1$.
Output: A random graph from $G(n, p)$.

$G \leftarrow \overline{K_n}$;
$V \leftarrow \{0, 1, \ldots, n-1\}$;
$E \leftarrow \{$2-combinations of $V\}$;
for *each* $e \in E$ **do**
    $r \leftarrow$ draw uniformly at random from interval $(0, 1)$;
    if $r < p$ **then**
        add edge e to G;
return G;

1.1 Research Questions

An introduction chapter will typically have a section providing a concise statement of the research question or questions as opposed to the general motivation of the area of research provided in the preceding introductory section.

1.2 Structure of the Dissertation

It is customary to include a section at the end of the introduction chapter providing a brief summary of the contents of each of the following chapters in a few sentences or at most 1–2 paragraphs.

Appendix A

Further Results

This chapter is intentionally left blank.

Nomenclature

$\mathcal{P}(A)$ The power set of A

DOI A system for persistent identification of digital content objects maintained by the International DOI Foundation.

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Bibliography

[1] BIGGS, N. L., LLOYD, E. K., AND WILSON, R. J. *Graph Theory 1736–1936*. Clarendon Press, Oxford, United Kingdom, 1976. 1

[2] GOEL, A., KAPRALOV, M., AND KHANNA, S. Perfect Matchings in $O(n \log n)$ Time in Regular Bipartite Graphs. In *Proceedings of the 42nd ACM Symposium on Theory of Computing (STOC '10)* (Cambridge, MA, USA, May 2010), ACM Press, pp. 39–46. doi:10.1145/1806689.1806697. 2

[3] KAWARABAYASHI, K.-I., AND KOBAYASHI, Y. Algorithms for finding an induced cycle in planar graphs. *Combinatorica 30*, 6 (Nov. 2010), 715–734. doi:10.1007/s00493-010-2499-x. 1

[4] WAGNER, J. D. *The Combinatorics of the Permutation Enumeration of Wreath Products between Cyclic and Symmetric Groups*. Ph.D. thesis, Department of Mathematics, University of California at San Diego, San Diego, CA, USA, May 2000. 1